Sparse Array Application in Sensing

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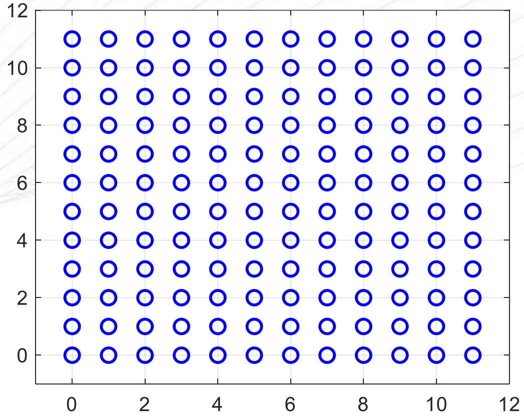
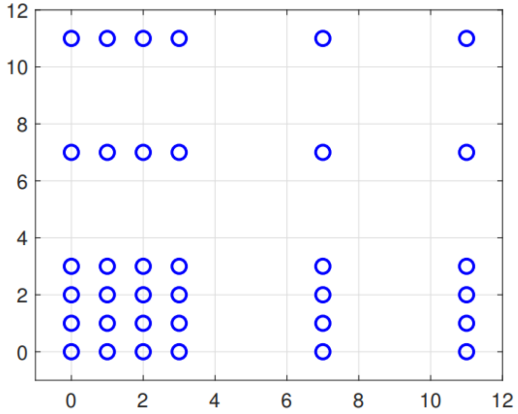
Issues of uniform array & benefits of sparse array

Sensing by uniform antenna array vs. sparse antenna array (spatial domain)



Uniform planar array (UPA)



Sparse planar array (SPA)

- Uniform antenna array (ULA, UPA)
  - To have high spatial resolution (DoA estimation precision) → antenna panel aperture should be sufficient large
  - To have large field of view (FOV) (i.e., to avoid alias beam or faulty detection) → inter-antenna interval should be sufficient small (≤ half-wavelength for 180º FOV)

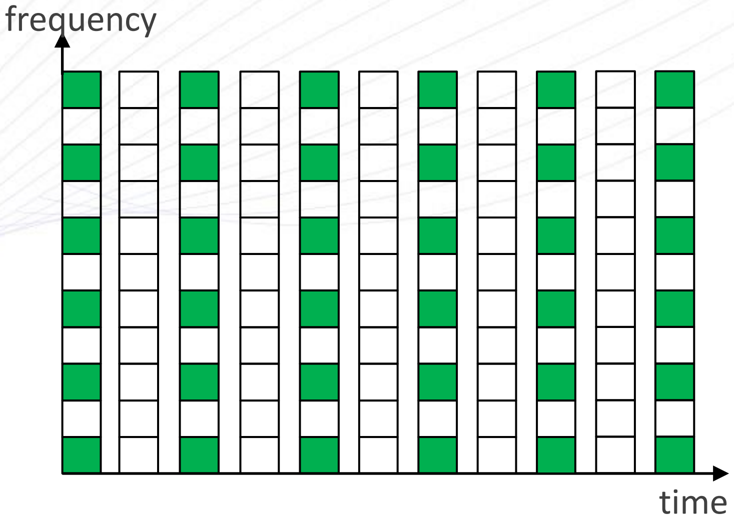
large amount of antennas → high hardware cost, high power consumption

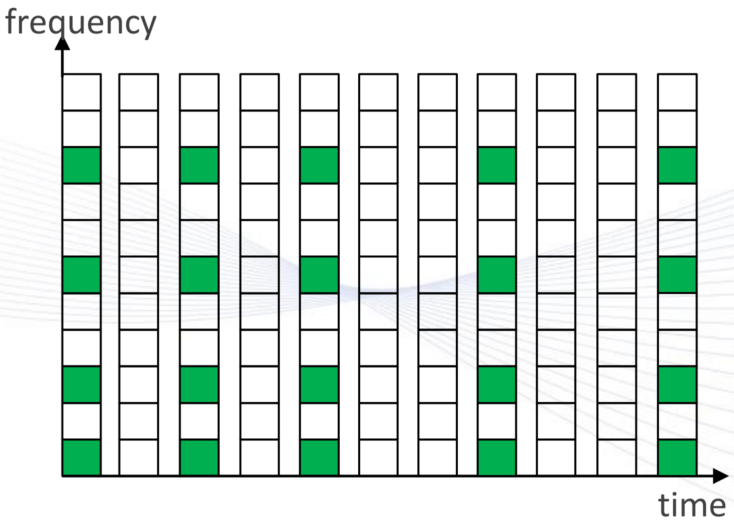
- Sparse antenna array (SLA, SPA)
  - To have the same spatial resolution → the same antenna panel aperture as uniform antenna array
  - To have the same field of view → the same minimum inter-antenna interval as uniform antenna array

small number of antennas → low hardware cost, low power consumption

Issues of uniform array & benefits of sparse array

Sensing by uniform pilot placement or sparse pilot placement (frequency/time domain)





- Uniform pilot placement
  - To have high delay/Doppler resolution (estimation precision) → sensing signal BW/duration should be sufficient large
  - To have large delay/Doppler estimation range → inter-pilot interval should be sufficient small

large amount of pilot overhead → low spectrum utilization ratio

- Sparse pilot placement
  - To have the same resolution (estimation precision) → the same sensing signal BW/duration as uniform pilot array
  - To have the same delay/Doppler estimation range → the same minimum inter-pilot interval as uniform pilot array

small amount of pilot overhead → high spectrum utilization ratio

Applications of sparse array

Sparse array can be considered in radar, sensing, imaging and wireless communication

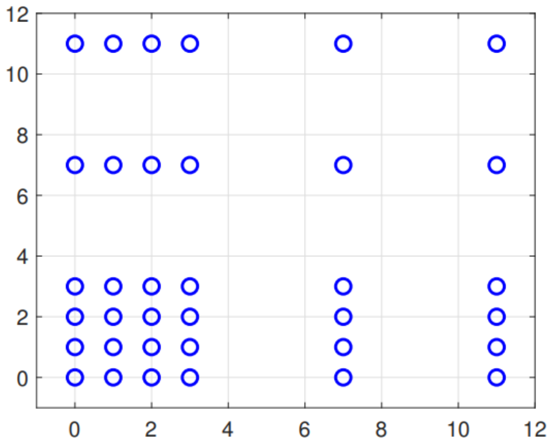
Capability of sparse array in sensing:

- Given N sensors, the difference co-arrays (DCA) form of these arrays have $O(N^2)$ elements, so it can identify $O(N^2)$ uncorrelated sources.

$$E\left(\boldsymbol{y}_i\boldsymbol{y}_j^H\right) = \frac{1}{N}\left(\sum_{n=1}^{N}\boldsymbol{y}_i^{(n)}\boldsymbol{y}_j^{(n),H}\right) \to 0,$$

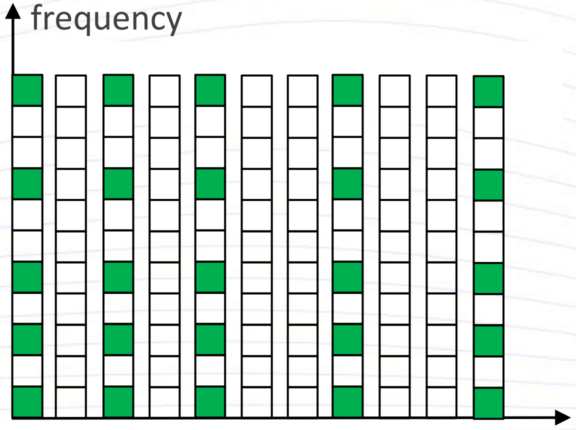
where $\boldsymbol{y}_i^{(n)}$ represents SD/FD/TD received signals for object i at multiple snapshots $n = 1, 2, \cdots$

Sparse antenna array (sparsity in spatial domain)



antenna array

Sparse pilot placement (sparsity in frequency/time domain)



pilot placement

Application areas

- DoA estimation
- Delay estimation
- Doppler estimation

Challenges of applying sparse array in sensing

- Sparse array construction
  - The optimal sparse array can only be derived by computer exploration / exhaustive searching (high computation) → impractical if array size is large
  - Practical sparse arrays: by closed-form formulas

- Sparse array-based sensing processing
  - Array elements in physical form are discontinuous → to obtain continuous array elements in Difference Co-Array (DCA) form
  - Need uncorrelated sources to generate covariance matrix, but the multi-path Rx signals (corresponding to multiple objects) are correlated → to rely on different channel parameters in multiple dimensions (spatial/frequency/time)

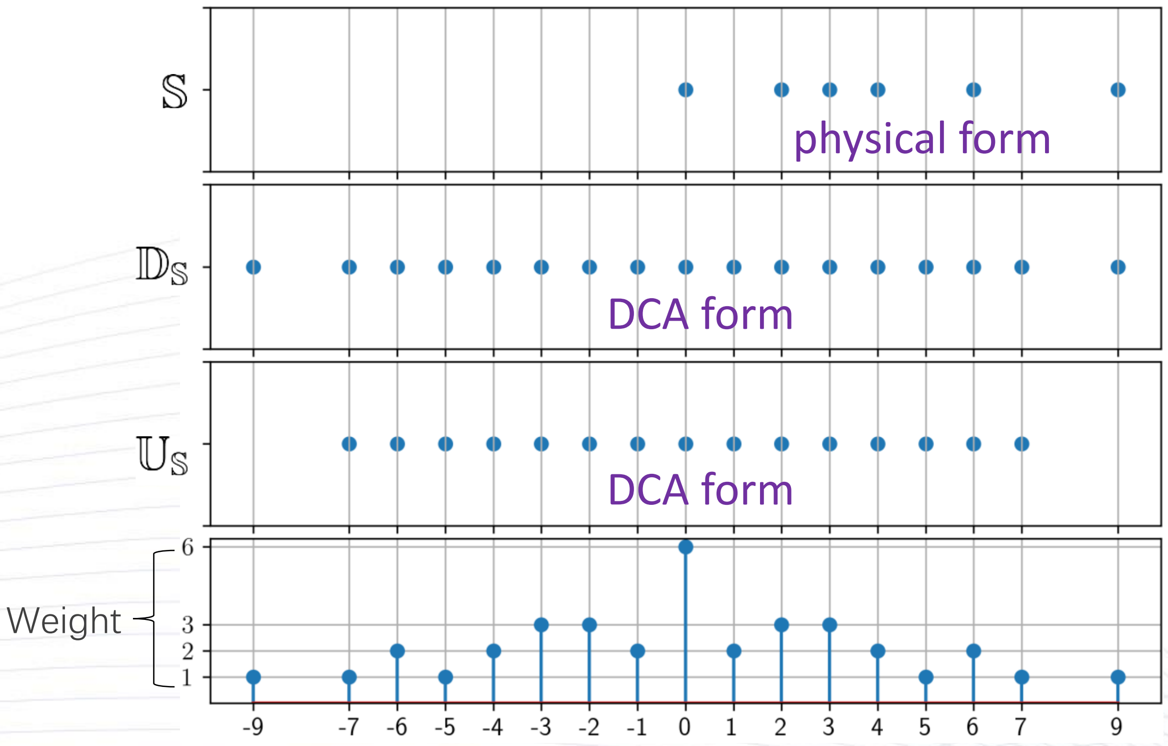
Sparse array construction

Difference Co-Array (DCA) form

- Sparse array
  - Array element position set $\mathbb{S} = \{s_i \mid i = 1,2,\dots,N\}$
- Difference co-array (DCA)
  - DCA is the difference between two antennas' positions.
  - For a certain sparse array, full DCA set $\mathbb{D}_\mathbb{S} = \{s_i - s_j \mid i,j = 1,2,\dots,N\}$
  - Spatial lag: an element of $\mathbb{D}_\mathbb{S}$
  - Degree of freedom (DoF): cardinality of $\mathbb{D}_\mathbb{S}$, i.e., $|\mathbb{D}_\mathbb{S}|$.
  - Weight of a spatial lag: number of antenna pairs with identical DCA value
- Effective DCA
  - For a certain sparse array, effective DCA set $\mathbb{U}_\mathbb{S}$ has continuous values; $\mathbb{U}_\mathbb{S} \subseteq \mathbb{D}_\mathbb{S}$
  - $\frac{|\mathbb{U}_\mathbb{S}|-1}{2}$ **uncorrelated** sources can be identified.

Example

- $\mathbb{S} = \{0, 2, 3, 4, 6, 9\}$
- $\mathbb{D}_\mathbb{S} = \{-9, -7, \cdots, 0, \cdots 7, 9\}$
- $\mathbb{U}_\mathbb{S} = \{-7, \cdots, 0, \cdots 7\}$
- $|\mathbb{D}_\mathbb{S}| = 17$
- $|\mathbb{U}_\mathbb{S}| = 15$
- 7 uncorrelated sources can be identified.



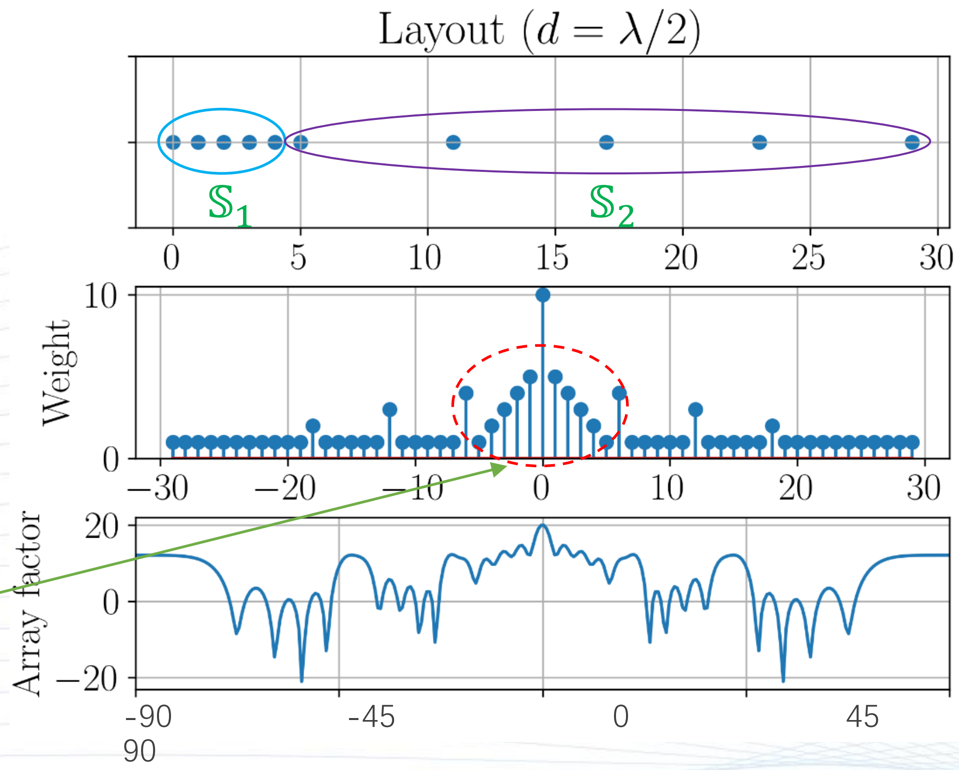
Sparse array construction

Formulated sparse array (1): nested array

- Antenna number: $N = N_1 + N_2$
  - For even N, $N_1 = N_2 = \frac{N}{2}$.
  - For odd N, $N_1 = \frac{N-1}{2}$, $N_2 = \frac{N+1}{2}$.
- Consist of 2 ULA: $\mathbb{S} = \mathbb{S}_1 \cup \mathbb{S}_2$
  - 1st ULA: $\mathbb{S}_1 = \{0, 1, \ldots, N_1 - 1\}$
  - 2nd ULA: $\mathbb{S}_2 = \{N_1 + (N_1 + 1)i \mid i = 0, 1, \ldots, N_2 - 1.\}$
- Advantages
  - Closed-form expression
  - Easy to generate
- Disadvantage
  - High redundancy at low spatial lags

Example

- N=10, N_1=N_2=5
- sparse(10): $\mathbb{S}$ = {0, 1, 2, 3, 4, 5, 11, 17, 23, 29}
- $\mathbb{D}_{\mathbb{S}} = \mathbb{U}_{\mathbb{S}} = \{-29, \cdots, 0, \cdots 29\}$



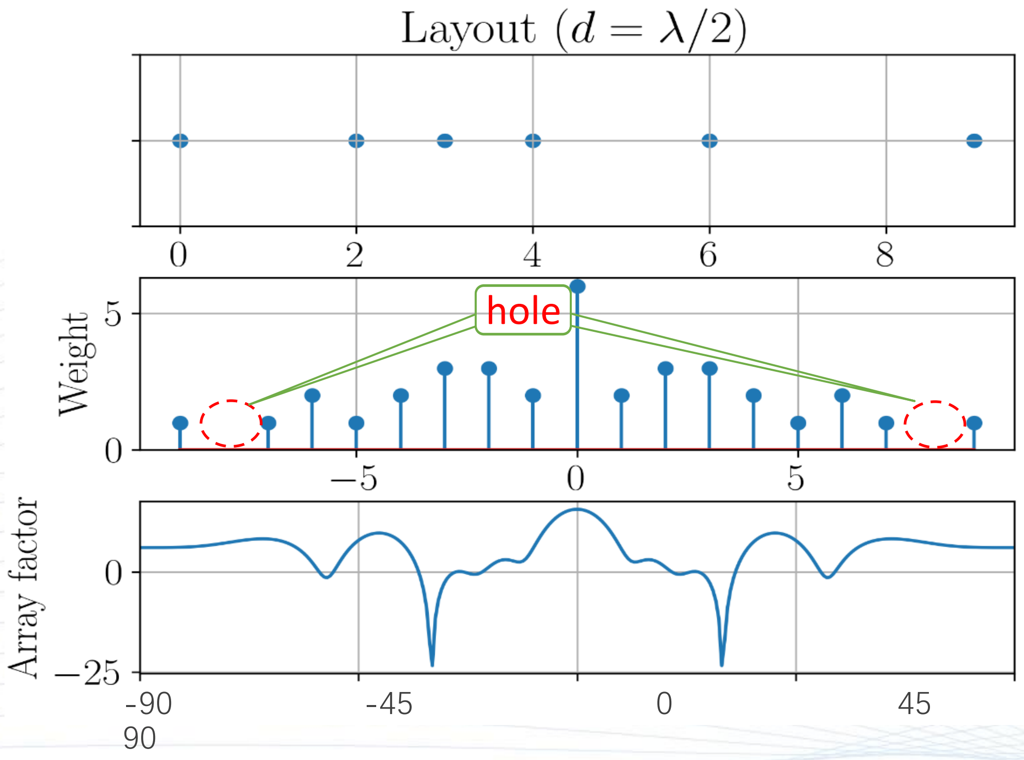
Sparse array construction

Formulated sparse array (2): Coprime array

- Antenna number: $N = 2P + Q - 1$
  - P and Q are coprime integers, $P < Q$
- Consist of 2 ULA: $\mathbb{S} = \mathbb{S}_1 \cup \mathbb{S}_2$
  - 1st ULA: $\mathbb{S}_1 = \{qP \mid q = 0, 1, \ldots, Q - 1\}$
  - 2nd ULA: $\mathbb{S}_2 = \{pQ \mid p = 0, 1, \ldots, 2P - 1\}$
- Advantages
  - Closed-form expression
  - Easy to generate
- Disadvantage
  - Not hole-free DCA: $|\mathbb{U}_{\mathbb{S}}| < |\mathbb{D}_{\mathbb{S}}|$

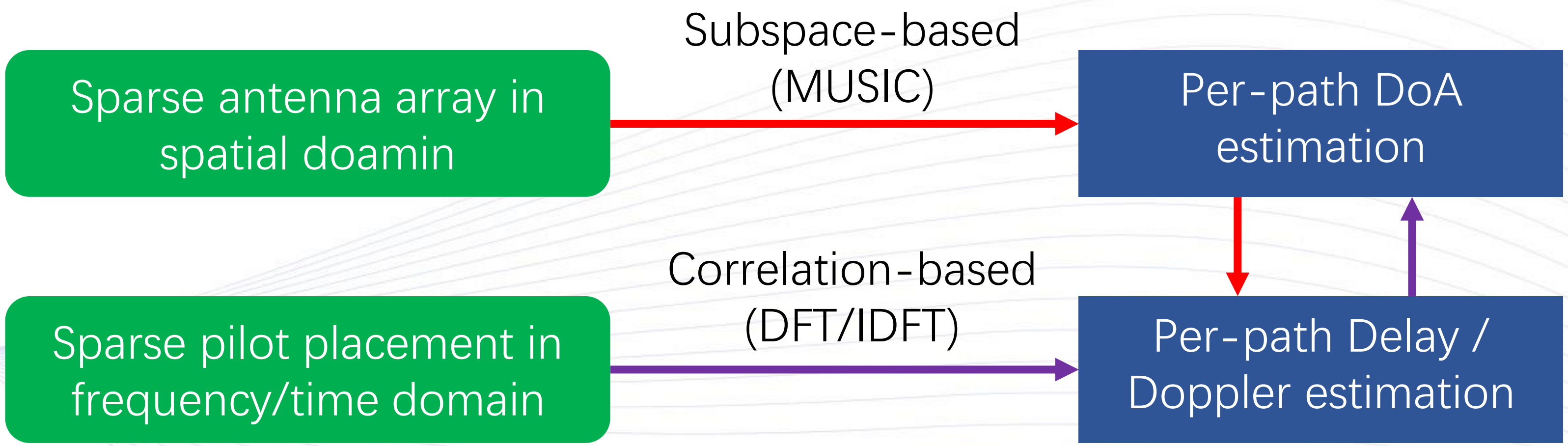
Example

- $P = 2$, $Q = 3$
- sparse(6): $\mathbb{S} = \{0, 2, 3, 4, 6, 9\}$
- $\mathbb{D}_{\mathbb{S}} = \{-9, -7, \cdots, 0, \cdots 7, 9\}$
- $\mathbb{U}_{\mathbb{S}} = \{-7, \cdots, 0, \cdots, 7\}$



Sparse array sensing algorithms

	Subspace-based (MUSIC)	Correlation-based (DFT/IDFT)
Complexity	high	low
Feasibility on array length	Fit to short array (e.g., spatial domain)	Fit to long array (e.g., frequency/time domain)
Reason	Using subspace-based algorithm on long array leads to high complexity.	Using DFT/IDFT-based algorithm on short array leads to low resolution.



Uncorrelation Criterion (1)

- When sensing signal is reflected by multiple objects, the received signal $\boldsymbol{y} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_L] \cdot [s_1, s_2, \cdots, s_L]^T + noise \triangleq \boldsymbol{As} + noise$, then the covariance matrix of the received signal $E(\boldsymbol{yy}^H) = \boldsymbol{A}E(\boldsymbol{ss}^H)\boldsymbol{A}^H + \sigma_n^2 \boldsymbol{I}$.
- To enable estimating steering vectors $\boldsymbol{A}$ by DCA-based processing with sparse array (antenna or pilot placement), it is required that the covariance matrix for object coefficients is a diagonal matrix, i.e., $E(\boldsymbol{ss}^H) = \mathrm{diag}\left(\sigma_{s,1}^2, \cdots, \sigma_{s,L}^2\right)$.
  - This is equivalent to requiring s_i and s_j are uncorrelated, i.e., $E(s_i s_j^*) = \frac{1}{K}\left(\sum_{k=1}^{K} s_i^{(k)} s_j^{(k),*}\right) = 0$, $\forall i, j$, where $s_i^{(k)}$ means the coefficient for object i in the kth snapshot. We name this requirement as **Uncorrelation Criterion** for sparse array.
  - For **angle** estimation, steering vector estimation is in SD; snapshots are derived in FD/TD domain.
  - For **delay** estimation, steering vector estimation is in FD; snapshots are derived in SD/TD domain.
  - For **Doppler** estimation, steering vector estimation is in TD; snapshots are derived in SD/FD domain.

Uncorrelation Criterion (2)

- As the transmitted sensing signal is common for all objects, to satisfy Uncorrelation Criterion, for sparse array-based parameter estimation in one dimension (SD, FD or TD), a <u>necessary</u> condition is: multiple objects' channel parameters in the snapshot dimension (other than measurement dimension) should be different.

Table: requirements for objects' parameters for sparse array-based sensing

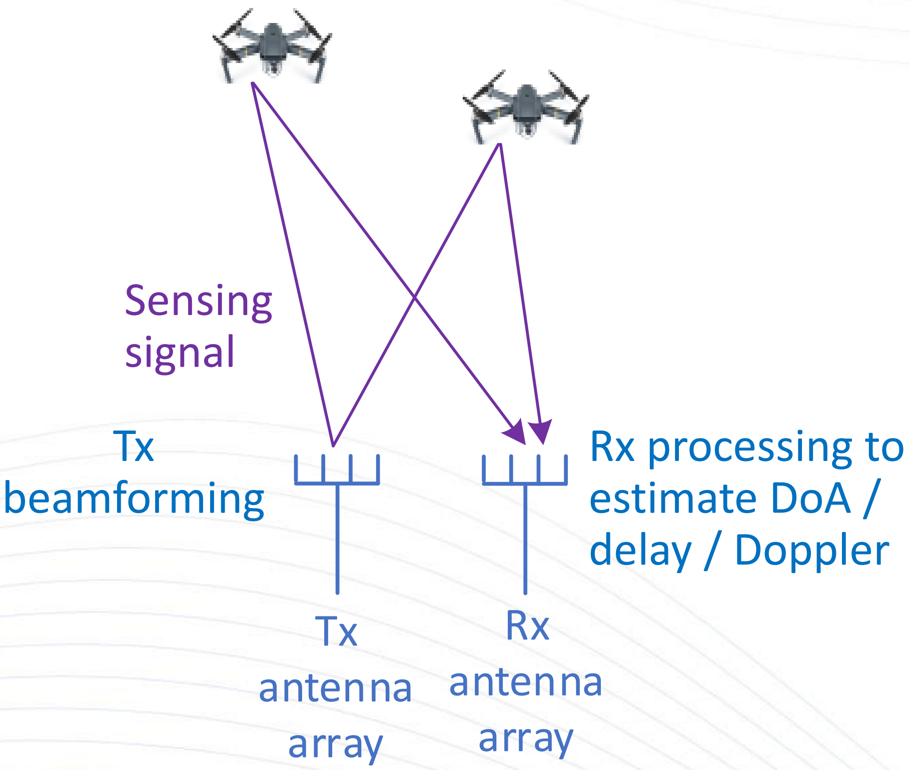
Sensing metrics	Measured parameter	Requirements for multiple objects' parameters
Object's direction	DoA	Different delays or Doppler frequencies
Object's distance	delay	Different DoA angles or Doppler frequencies
Object's speed	Doppler frequency	Different DoA angles or delays

In UAV sensing, two-dimension snapshots increase the probability of different channel parameters between two objects.

Performance evaluation

Simulation scenario parameters (1)

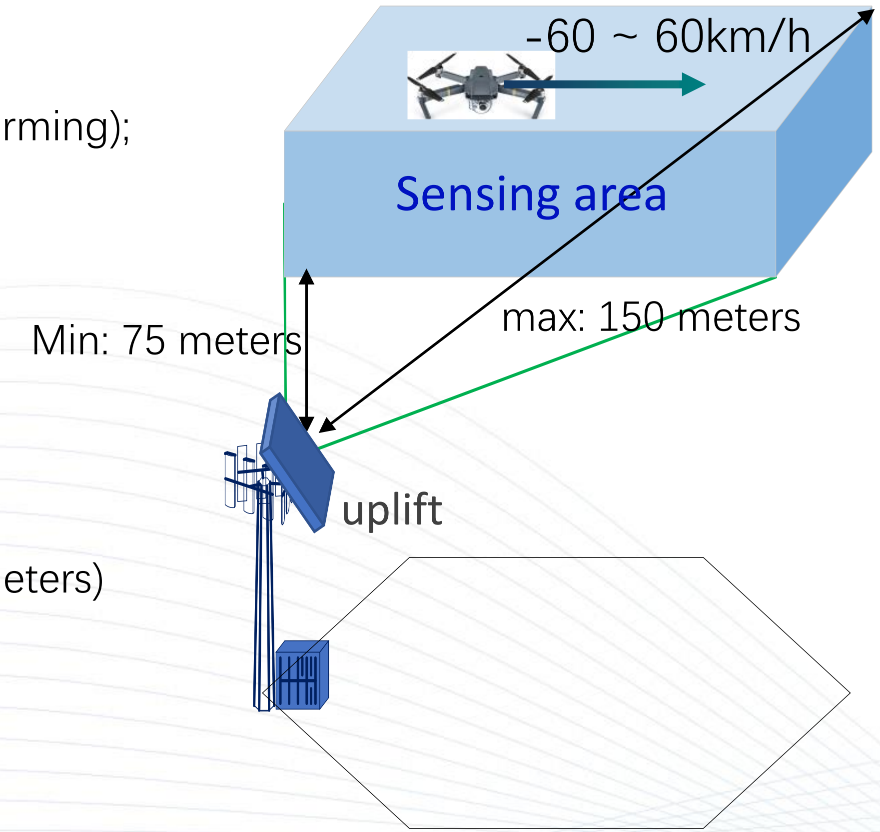
- Scenario
  - gNB senses UAVs (without communication to them)
- Sensing operations
  - Mono/bi-static sensing: the transmitted signal is reflected by one or multiple objects and then is received by sensing receiver.
  - Sensing signal receiver estimates DoA, delay and/or Doppler frequency for the main arrived path(s), each possibly corresponding to one UAV.



Performance evaluation

Simulation scenario parameters (2)

- Channel parameters
  - Channel model: LOS path + NLOS paths with K = 15dB
  - Sensing Rx power $P_{r,LOS} = \frac{\sigma P_t G_t G_r \lambda^2}{(4\pi)^3 d^4}$
    - P_t = 46dBm; G_t = 17dB (w/ Tx beamforming), G_r = 8dB (w/o Rx beamforming);
    - UAV RCS σ = 0.1m^2
    - Noise spectrum density = -174dBm/Hz, noise figure = 5dB
- Sensing area
  - DoA
    - relative to the uplift gNB panel: random in $\theta \in (0, 90°)$, $\varphi \in (0, 360°)$
  - Delay
    - Tx-object-Rx: random in 500ns (height 75 meters) ~ 1μs (height 150 meters)
  - Speed
    - random in -60 ~ 60km/h and random direction

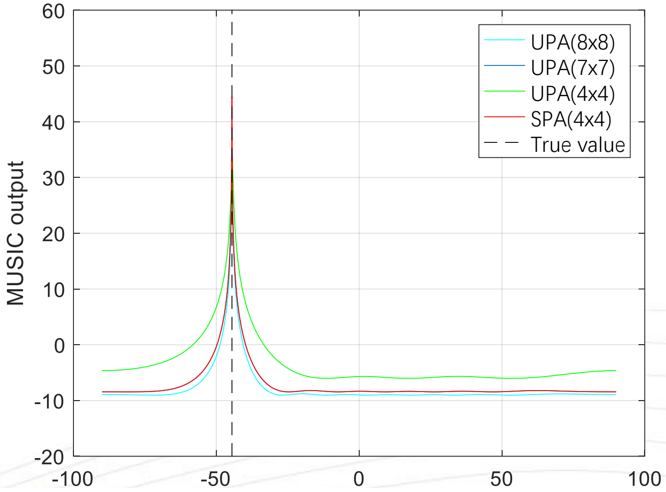


Performance evaluation on sparse antenna array

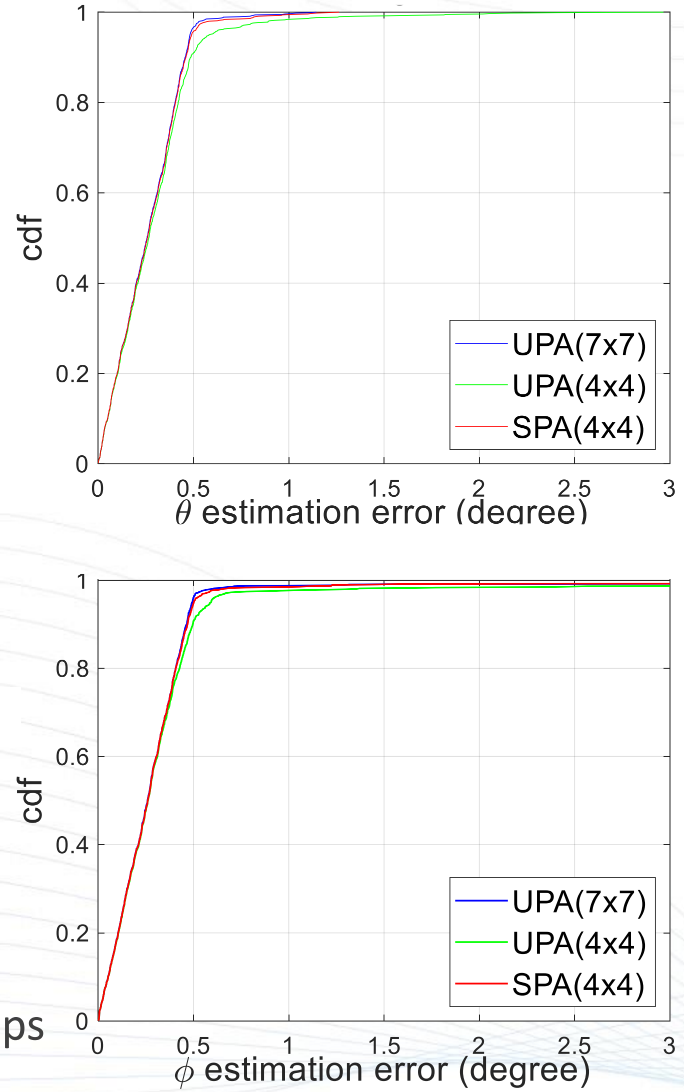
DoA estimation of one object

- Algorithm
    - MUSIC-based
- Observation
    - UPA(8) has the best performance.
        - Reason: UPA(8) has the largest antenna array aperture size.
    - SPA(4) is slightly weaker than UPA(7).
        - Reason: SPA(4) has the same antenna aperture size as UPA(7), but less antennas than UPA(7) → SPA(4) has loss in SNR than UPA(7).
    - SPA(4) is obviously better than UPA(4).
        - Reason: UPA(4) has the smallest antenna array aperture size.

MUSIC output for single UAV drop



DoA angle (degree) for horizontal antenna array

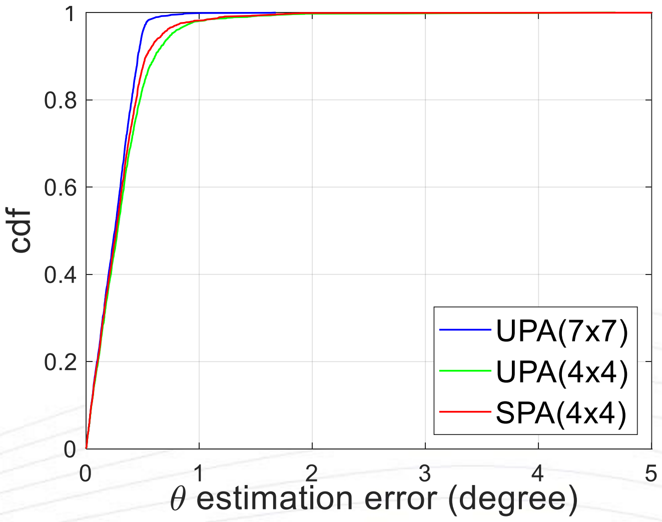


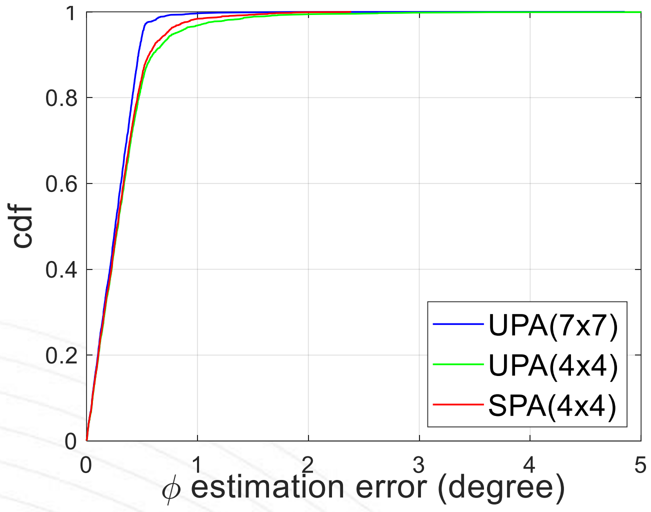
cdf of DoA (elevation, azimuth) estimation errors for multiple UAV drops

Performance evaluation on sparse antenna array

DoA estimation of two objects

- Algorithm
  - MUSIC-based
- Observation
  - MD/FA of SPA(4) is similar as UPA(7), lower than UPA(4).
  - Accuracy of SPA(4) is weaker than UPA(7) @ 20% tile, slightly better than UPA(4).
  - Reason:
    - Uncorrelation Criterion for DoA estimation by sparse array depends on the differences of two objects' delays or Doppler frequencies.
    - When these differences are zero or close to zero, the Uncorrelation Criterion isn't or is weekly satisfied, then leads to performance degradation.



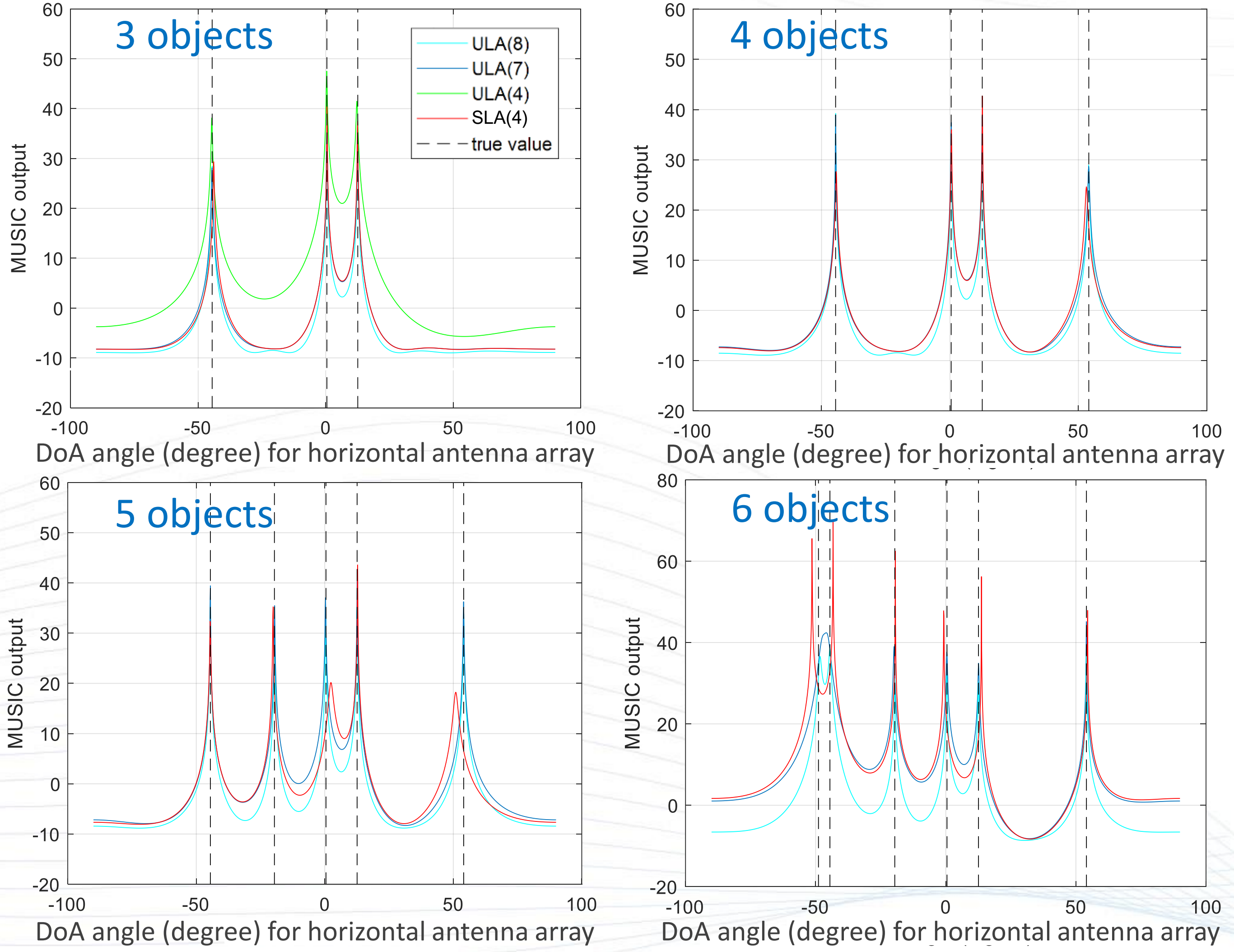


Sensing of two objects	UPA(7x7)	UPA(4x4)	SPA(4x4)
Miss-detection (MD) (%)	0.65	1.65	0.7
False-alarm (FA) (%)	0.5	2	0.4

Performance evaluation on sparse antenna array

DoA estimation of three ~ six objects

- Algorithm: MUSIC
- Observation
  - ULA(4) can detect up to 3 objects. Sparse(4) and ULA(7) can detect up to 6 objects.
    - Reason: SLA(4) has continuous 7 elements in DCA form.
  - SLA(4)'s accuracy is lower than ULA(7/8), and performance loss gets higher with the increase of object number.
    - Reason: in practice, the correlative coefficient $\rho_{i,j} \neq 0$, so the more objects, the larger is the accumulated impact of non-zero correlative coefficients.



Sparse array usages in sensing

- Sensing scenarios with single object
  - Performance of sparse array is close to uniform array when SNR is high enough.
  - Sparse antenna array or sparse pilot placement can be adopted to estimate the DoA/delay/Doppler of the object.

Sparse array can be used for fine sensing (high-accuracy localization/tracking) with single object

- Sensing scenarios with multiple objects
  - If without sufficiently-large differences in DoA/delay/Doppler, sparse array may under-perform (with higher MD/FA ratio and lower accuracy) uniform array.

Sparse array can be used for coarse sensing (existence detection or low-accuracy localization/tracking) with multiple objects

  - Sparse antenna array can support the maximum countable number of objects as uniform antenna array with the same antenna aperture size.

Sparse array can be used for counting the objects' quantity

THANKS